Measurement & Control

Measurement Fundamentals

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Measurement Fundamentals

Definitions & Dimensions

DIMENSIONS

All the derived units in mechanics can be expressed in terms of the fundamental units of length, mass and time. For example,

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{\text{Mass}}{(\text{Length})^3} = \frac{(\text{M})}{(\text{L})^3} = (\text{ML}^{-3})$$

$$\text{Velocity} = \frac{(\text{L})}{(\text{T})} = (\text{LT}^{-1})$$

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time}} = \frac{(\text{L})}{(\text{T})(\text{T})} = \frac{(\text{L})}{(\text{T})^2} = (\text{LT}^{-2})$$

L, M and T denote the dimensions of length, mass and time, respectively. The powers to which the fundamental units are raised to obtain the derived units are called the *dimensions of the derived units.* Volume has three dimensions in Length (L^3). The dimensions of density are one in mass and L^{-1} in length and are written as ML^{-3}. Usually, the dimensions of quantities are expressed as equations as shown above. Such equations are called *dimensional equations*. Some quantities like angle and indices are dimensionless quantities. Numerical quantities have no dimensions. Quantities like specific gravity, which are expressed in the form of a ratio of two quantities having the same units, are dimensionless. The dimensions of a quantity will be the same whatever be the units in which it is measured.

The dimensional equations help (i) in conversion from one system of units to another one (ii) in derivation of equations for physical quantities and (iii) in checking the accuracy of an equation.

DIMENSIONS IN ELECTROSTATIC(ES) AND ELECTROMAGNETIC(EM) SYSTEMS

According to Coulomb's inverse square law, force between two charges Q_1 and Q_2 placed at a distance *d* from each other in a medium of absolute permittivity $\in$ is given by

$$F = \frac{Q_1 Q_2}{\varepsilon d^2}$$

Dimensionally,

$$[\text{MLT}^{-2}] = \frac{[\text{Q}^2]}{[\varepsilon \text{L}^2]}$$

$\therefore$ $[Q] = [\in^{1/2} M^{1/2} L^{3/2} T^{-1}]$

Also force between two magnetic poles of strength m_1 and m_2 placed at distance d from each other in a medium of absolute permeability μ is given by

$$F = \frac{m_1 m_2}{\mu d^2}$$

Dimensionally

$$[MLT^{-2}] = \frac{[m^2]}{[\mu L^2]}$$

∴ $$[m] = [\mu^{1/2} M^{1/2} L^{3/2} T^{-1}]$$

(i)

From the above two dimensional expressions, one for electrical and other for magnetic quantity it is obvious that the dimensions of these two quantities involve the dimensions of either ∈ or μ in addition to those of length, mass and time. It holds good for all such quantities.

RELATION BETWEEN (M) AND (∈)

Since force exerted upon a magnetic pole of strength *m* units placed at the centre of a circular wire of radius *r* and carrying a current *i*, in an arc of the circle of length *l* is given by

$$F = \frac{mil}{r^2}$$

Or $$i = \frac{Fr^2}{ml}$$

And $$Q = it = \frac{Fr^2}{ml}.t \qquad (ii)$$

Substituting dimensions of Q and m from expressions (i) and (ii) we get

$$[\varepsilon^{1/2} L^{3/2} M^{1/2} T^{-1}] = \frac{[MLT^{-2}][L^2][T]}{[\mu^{1/2} L^{3/2} M^{1/2} T^{-1}][L]}$$

Or $$[\varepsilon^{1/2} L^{3/2} M^{1/2} T^{-1}] = [M^{1/2} L^{1/2} \mu^{-1/2}]$$

Or $$[\mu^{-1/2} \varepsilon^{1/2}] = [LT^{-1}]$$

Or $$\frac{1}{[\mu\varepsilon]^{1/2}} = [LT^{-1}] = \text{Dimensions of a velocity} \qquad (iii)$$

In any system of units the permeability and permittivity of free space (μ_o and $\in_o$) are related by the expression.

$$\mu_0 \varepsilon_0 = \frac{1}{C^2}, \text{ Where C is the velocity of light.}$$

From the above relation, the dimensions of any electrical quantity can be converted from those of the *e.m.u.* System of *e.s.u.* System and vice versa.

DIMENSIONS OF ELECTRICAL AND MAGNETIC QUANTITIES

The dimensions of various electrical and magnetic quantities can be derived from the known relationship between them, as given below:

(i) Current

We know that magnetizing force at the centre of a loop of radius r is

$$H = \frac{2\pi I}{r}$$

or $$[H] = \frac{[I]}{[L]}$$

$\therefore$ Dimensions of current are

$$[I][H][L] = [\mu^{-1/2} M^{1/2} L^{1/2} T^{-1}][T] = [\mu^{-1/2} M^{1/2} L^{1/2}]$$

(ii) Charge

Since quantity of electricity = current x time

$\therefore$ $[Q] = [TI]$

(iii) Potential Difference

Since potential difference = work done/Quantity of electricity

$$\therefore \quad [V] = \frac{[E]}{[Q]} = \frac{[ML^2T^{-2}]}{[TI]} = [ML^2T^{-3}I^{-1}]$$

(iv) Resistance

Since by Ohm’s law

Resistance = potential difference/current

$$\therefore \quad [R] = \frac{[V]}{[I]} = \frac{[ML^2T^{-3}I^{-1}]}{[I]} = [ML^2T^{-3}I^{-2}]$$

(v) Magnetic flux

Since e.m.f. = Rate of change of flux

$$\therefore \quad V = \frac{[\phi]}{[T]}$$

or $$[\phi] = [V][T] = [ML^2T^{-3}I^{-1}][T] = [ML^2T^{-2}I^{1}]$$

(vi) Magneto motive Force

Dimensions of m.m.f. are identical with those of current since m.m.f. can be defined by current-turns

[M.M.F.] = [I]

(vii) Inductance

Since e.m.f. = inductance x rate of change of current

$$\therefore \quad [\text{Inductance}] = \frac{[ML^2T^{-3}I^{-1}]}{[IT]^{-1}} = [ML^2T^{-2}I^{-2}]$$

(viii) Electric Field Strength

Since electric field strength = Potential gradient

$$\therefore \quad [E] = \frac{[V]}{[L]} = \frac{[ML^2T^{-3}I^{-1}]}{[L]} = [MLT^{-3}I^{-1}]$$

(ix) Capacitance

Since capacitance = charge/potential

$$\therefore \quad [C] = \frac{[Q]}{[V]} = \frac{[TI]}{[ML^2T^{-3}I^{-1}]} = [M^{-1}L^{-2}T^4I^2]$$

(x) Permeability (μ)

Flux = flux density x area = $[I^{-1}MT^{-2}][L^2] = [I^{-1}ML^2T^{-2}]$

MMF = turns x current = $[I]$

Reluctance = MMF/Flux = $\frac{[I]}{[I^{-1}ML^2T^{-2}]} = [I^2M^{-1}L^{-2}T^2]$

Now Permeability = length/(reluctance x area) = $\frac{[L]}{[I^2M^{-1}L^{-2}T^2][L^2]} = [I^{-2}MLT^{-2}]$

(xi) Conductivity

= 1/resistivity = $[I^2M^{-1}L^{-3}T^3]$

(x) Pole Strength

Force $F = \frac{m_1m_2}{\mu d^2}$

where d is distance between poles of strength m_1 and m_2. Writing the dimensions, we have

$$[MLT^{-2}] = \frac{[m^2]}{[\mu][L^2]} = [\mu^{1/2}M^{1/2}L^3T^{-1}]$$

Example

Show that $\omega^2 LC$ is non-dimensional, ω being the angular frequency of the applied voltage.

Solution

$$[\omega] = [T^{-1}]$$

$$[L] = [ML^2 T^{-2} I^{-2}]$$

$$[C] = [M^{-1} L^{-2} T^4 I^2]$$

$$[\omega^2 LC] = [\omega^2][L][C]$$

$$= [T^{-2}][ML^2 T^{-2} I^{-2}][M^{-1} L^{-2} T^4 I^2] = [M^o L^o T^o I^o]$$

Hence ω^2 LC is non-dimensional quantity.

Example

The expression for mean torque of an electro-dynamometer type wattmeter may be written as : $T \propto M^a E^b Z^c$

where T is torque, M is mutual inductance, E is the voltage and Z is impedance. Determine the values of constants a, b and c by dimensional analysis. Use length, mass, time and current [L, M, T, I] as fundamental dimensions.

Solution

Let $T = K M^a E^b Z^c$ (i)

where k is a dimensionless constant number.

Dimensionally we can write

$$[T] = [ML^2 T^{-2}]$$

$$[M] = [ML^2 T^{-2} I^{-2}]$$

$$[E] = [ML^2 T^{-3} I^{-1}]$$

$$[Z] = [ML^2 T^{-3} I^{-2}]$$

Substituting the dimensions of quantities involved in expression (i) we get

$$[ML^2 T^{-2}] = [ML^2 T^{-2} I^{-2}]^a [MT^2 T^{-3} I^{-1}]^b [ML^2 T^{-3} I^{-2}]^c$$

or $[ML^2 T^{-2}] = [M^{a+b+c} L^{2a+2b+2c} T^{-2a-3b-3c} I^{-2a-b-2c}]$

Equating the corresponding indices, we have

For M $a + b + c = -1$ (ii)

For T $-2a -3b -3c = -2$ (iii)

For I $-2a -b -2c = 0$ (iv)

Solving expressions (ii), (iii) and (iv) we get a =1; b = 2 and c = -2 **Answer**

Hence the expression is

$$T \propto ME^2 Z^{-2}$$

Or $$T \propto \frac{ME^2}{Z^2}$$ **Answer**

Example

Show that the product $(\mu\varepsilon)^{-1/2}$ has the dimension of velocity.

Solution

Velocity is defined as distance per second.

$$\text{Velocity} = \frac{\text{length}}{\text{time}} = \frac{[L]}{[T]} = [LT^{-1}]$$

When we use e.m. and e.s. system of units, the dimensional equations are different for the same quantity. Any one quantity should have the same dimensions whatever may be the system. The dimension of charge is

e.m.u. system are

$$[Q] = [\mu^{-1/2}M^{1/2}L^{1/2}]$$

and in e.s.u. system are

$$[Q] = [\varepsilon^{1/2}M^{1/2}L^{3/2}T^{-1}]$$

$\therefore$ equating the two equations, we have

$$[\mu^{-1/2}M^{1/2}L^{1/2}] = [\varepsilon^{1/2}M^{1/2}L^{3/2}T^{-1}]$$

$$[\mu^{-1/2}\varepsilon^{-1/2}] = [LT^{-1}]$$

Now $[LT^{-1}]$ are the dimensions of velocity, hence dimensions of velocity are

$$[LT^{-1}] = [\mu^{-1/2}\varepsilon^{-1/2}]$$

Example (AMIE Winter 08, 12 marks)

Derive the dimensions of potential difference in the electrostatic system in terms of mass, length and time. in the course of a calculation, an expression of the following form was arrived at:

$$I = E\left\{\frac{I}{Z_1} + \frac{j\omega M}{Z_2}\left(\frac{1}{R} + \frac{C}{L}\right)\right\}$$

Show that there must have been an algebraic error and point out the term(s) which require correction.

Solution

The dimensions of various quantities in e.m. system are:

Current $[I] = [\mu^{1/2} L^{3/2} M^{1/2} T^{-2}]$

Impedance $[Z] = [\mu L T^{-1}]$

Mutual Inductance

$$[M] = [\mu L]$$

Capacitance $[C] = [\mu^{-1} L^{-1} T^{2}]$

Resistance $[R] = [\mu L T^{-1}]$

Inductance $[L] = [\mu L T^{-1}]$

The left hand side term is dimensionally expressed as

$$[I] = [\mu^{-1/2} L^{1/2} M^{1/2} T^{-1}]$$

The right hand term $E\left\{\frac{I}{Z_1} + \frac{j\omega M}{Z_2}\left(\frac{1}{R} + \frac{C}{L}\right)\right\}$ is dimensionally expressed as

$$= [\mu^{1/2} L^{3} M^{1/2} T^{-2}]\, x \left\{\frac{1}{[\mu L T^{-1}]} + \frac{[T^{-1}][\mu L]}{[\mu L T^{-1}]}\left(\frac{1}{[\mu L T^{-1}]} + \frac{[\mu^{-1} L^{-1} T^{2}]}{[\mu L]}\right)\right\}$$

$$= [\mu^{1/2} L^{3} M^{1/2} T^{-2}]\, x \left\{[\mu^{-1} L^{-1} T] + ([\mu^{-1} L^{-1} T] + [\mu^{-2} L^{-2} T^{2}])\right\}$$

$$= \{[\mu^{-1/2} L^{1/2} M^{1/2} T^{-1}] + [\mu^{-1/2} L^{1/2} M^{1/2} T^{-1}] + [\mu^{-3/2} L^{-1/2} M^{1/2} T^{0}]\}$$

In order that the two sides should balance dimensionally, the dimensions of right hand side should be $[\mu^{-1/2} L^{1/2} M^{1/2} T^{-1}]$. We find that the dimensions of all the terms on the right hand side are not $[\mu^{-1/2} L^{1/2} M^{1/2} T^{-1}]$ and hence there is an algebraic error. The error is due to last term having dimensions of $[\mu^{-3/2} L^{-1/2} M^{1/2} T^{0}]$. The error can be eliminated only if it has the dimensions of $[\mu^{-1/2} L^{1/2} M^{1/2} T^{-1}]$. In order to get this, we must multiply the last term (C/L) by a quantity which has the dimension

$$\frac{[\mu^{-1/2} L^{1/2} M^{1/2} T^{-1}]}{[\mu^{-3/2} L^{-1/2} M^{1/2} T^{0}]} = [\mu L T^{-1}]$$

Standards

INTERNATIONAL & WORKING STANDARDS

The *international standards* are defied by international agreement. They represent certain units of measurement to the closest possible accuracy that production and measurement technology allow. International standards are periodically evaluated and checked by absolute measurements in to the fundamental units. These standards are maintained at the International Bureau of Weights and Measures and are not available to the ordinary user of measuring instruments for purposes of comparison or calibration.

Working standards are the principal tools of a measurement laboratory. The are used to check and calibrate general laboratory instruments for accuracy and performance or to perform comparison measurements in industrial applications. A manufacturer of precision resistances, for example, may use a standard resistor (a working standard) in the quality control department of his plant to check his testing equipment. In this case, he verifies that his measurement setup performs within the required limits of accuracy.

PRIMARY AND SECONDARY STANDARDS

The term standard' is applied to a piece of equipment, having a known measure of some physical quantity, which can be used, normally by a comparison method, for the purpose of obtaining the values of the physical of other equipment.

An absolute standard is one, whose value can be calculated directly from its physical dimensions only. Primary standards are absolute standards of such high accuracy that they can be employed as the ultimate reference standards for all electrical equipment. They are few in number and restricted to the various international standardizing laboratories. Apart from being highly accurate, they must be very stable i.e. their values must vary as little as possible over long periods of time and weather conditions.

Salient Features Of Primary And Secondary Standards

Primary standards

- These are absolute standards having highest degree of accuracy and are used as the ultimate reference standards.
- These are maintained by National Standard Laboratories in different parts of the world.
- They are independently calibrated by absolute methods at each of national laboratory periodically.
- These are not available for use outside the national laboratories.
- They are used in the calibration of secondary standards.
- These are highly stable over long periods of time .

- Primary standards are very few in number.

Secondary Standards

- There is reference standards use in industrial and day-to-day works.
- These are periodically calibrated and compared against primary standards. National Laboratories issue a certificate as regard their measured values in terms of primary standards.
- Responsibility of maintenance and calibration of these standards lies with the particular industry involved.

CAMPBELL PRIMARY AND SECONDARY STANDARDS OF MUTUAL INDUCTANCE (M)

The Campbell type of primary standard consists of a marble cylinder with screw threads carrying a primary coil of bare copper wire wound under tension and a marble ring with secondary coil placed in a channel cut in its circumference .

The primary coil is wound in single layer and is divided into two equal parts, which are connected in series and are displaced from one another by a distance equal to three times axial length of one of them.

The secondary coil is wound in a number of layers and is so placed that it is concentric coaxial with the primary coil cylinder. This coil is placed midway between the two portions of the primary coil and a means to bring the secondary coil in correct position relative to primary coil is provided. The mutual inductance, in case of above arrangement, will be maximum if radius of secondary coil is 1.46 times that of the primary coil.

Secondary standards of mutual inductance are used as standards of mutual inductance for use in laboratory. Such standards are constructed of values, which are either multiplies or fractions of the inductance of the primary standard. The values of a such a standard is determined by comparison with primary standard.

PRIMARY STANDARDS FOR TIME AND FREQUENCY

Time

The primary standard for time is second. infact any phenomenon that repeats itself can be used as the measure of time. The measurement consists of counting these repetitions. as oscillating pendulum, coiled spring as quartz crystal can be used for the purpose. Rotation of earth on its axis, which determines the length of a day is being used as a time standard from earliest time. One second being defined to be 1/86400 of mean solar day. This time defined in terms of earth rotations is called Universal Time. Further second was redefined in 1956 for scientific purpose requiring high precision as the fraction 1/31,556,925 9747 of the tropical year 1900. This time defined in terms of earths orbital motion is called Ephemeris Time(E.T.).

Frequency

The primary standard for frequency is Hertz. For the repeating motions time period ' T' is defined as the time required to complete one trip of the motion; that is, one complete oscillation or cycle. The frequency of this motion denoted by 'v' is the number of oscillations or cycles per unit of time. The frequency is therefore the reciprocal of the time period; or

$$v = 1/T$$

The MKS unit of frequency is the cycle per second as Hertz.

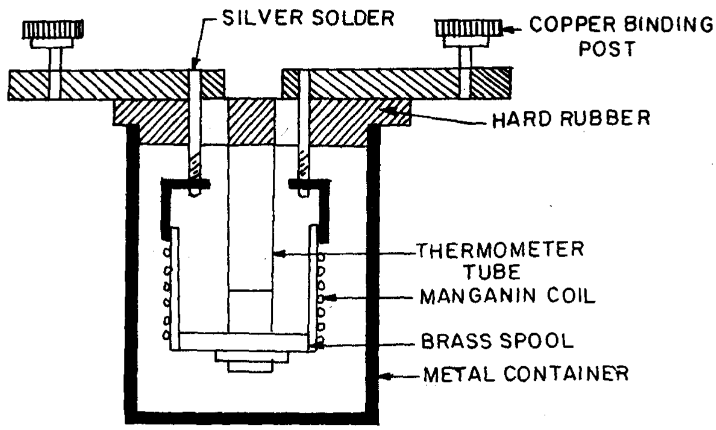
PRIMARY AND SECONDARY STANDARDS OF SELF INDUCTANCE (L)

Primary standards of self-inductance are rarely prepared as their value cannot be determined with an accuracy with which the values of primary standards of mutual inductance can be determined. Also the formula necessary for the calculation of the exact value of inductance for primary standards of self-inductance is very much complicated as compared with corresponding formula for mutual inductance. Hence primary standards of mutual inductance are usually considered as primary standards of self-inductance.

Secondary standards of self-inductance are the coils made of silk covered stranded copper wire wound on a marble former. These coils are made for nominal capacities of 0.0001, 0.001, 0.01, 0.1 and 1 Henry. In designing and constructing these standards all those precautions are taken which are taken in designing and constructing corresponding standards of mutual inductance.

WORKING AND CONSTRUCTIONAL DETAILS OF A PRIMARY STANDARD RESISTANCE

The standard resistor is coil of wire made of manganin mounted on a silk insulated brass spool. The coil uses a bifiler type of arrangement so as to reduce the residual inductance. After being wounded, resistance coils are impregnated with a shellac varnish and given an accelerated ageing by baking at a temperature of 150^0C for about 48 hours. Ageing makes the coil resistance stable with time. The coil is supported from the hard rubber top by means of a thermometer tube in which a thermometer can be inserted from outside. The copper lead wires, which are silver soldered to the ends of resistance coil are soft soldered to copper binding posts. The hard rubber top is screened into the metal container, which is filled with good quality of light mineral oil. Following figure shows the constructional details.



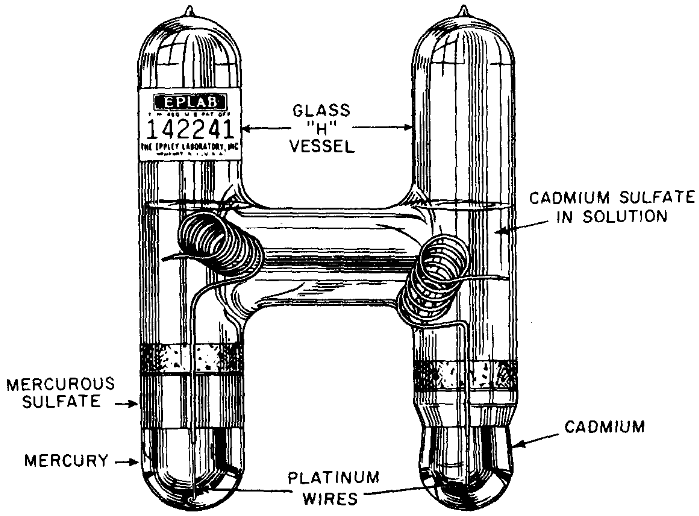
VOLTAGE STANDARDS - WESTON STANDARD CELL

The Weston standard cell is used as a standard of e.m.f. The e.m.f. of an ordinary cell depends on the materials employed and changes with the change in temperature and while supplying load, therefore, ordinary cells can not be taken as a source of constant e.m.f. for electrical measurements and other precision work.

The Weston cell is such a cell, whose e.m.f. remains constant for a longer period provided no appreciable current is drawn from the cell, therefore, such cells are never used as a source of energy but are employed as a secondary standard of voltage for electrical measurements.

Its e.m.f. when constructed in accordance with the standard specification, it 1.01824 International volts or 1.01859 absolute volts at 20^{o}C. It possesses the advantage of being reasonably robust and of having a small and accurately known temperature co-efficient of e.m.f. (Its e.m.f. falls 40 μ V per ^{o}C rise in temperature).

Weston standard cell is shown in fig. 1.1. Its consists of H-shaped glass vessel, each leg being 25 m.m. in diameter containing saturated solution of cadmium sulphate ($CdSO_4$) in the upper part, then crystals of cadmium sulphate, the paste of mercurous sulphate and cadmium sulphate over the mercury at the bottom of one leg and crystals of cadmium sulphate and cadmium amalgam at the bottom of second leg. The two limbs of the vessel are hermetically sealed. Cadmium sulphate solution acts as an electrolyte, mercury acts as +ve electrode, cadmium amalgam (a solution of 1 part of cadmium in 7 parts of mercury) acts as a -ve electrode and paste of mercurous sulphate and cadmium sulphate acts as a depolariser. Crystals of cadmium sulphate are put in the cell inorder to keep the electrolyte saturated. The connections of the cell to the external circuit are made by platinum wires sealed into the glass.



Precautions in the Use of Standard Cells

- The cell gets damaged and e.m.f. does not remain constant if a current of more than a few micro-amperes it drawn from it, therefore, care should be taken and current drawn from the cell should be kept to a minimum by protective resistance in the standard cell circuits and by proper manipulation of keys.
- Laboratory conditions, which would results in the condensation on moisture of the cell case should be avoided.
- Care should be taken also while moving the standard cell, as an appreciable shaking up of chemicals in the cell tends to cause variation in its e.m.f.
- The standard cells should be kept at dry places having uniform temperature of about 15^oC to 20^oC for the purpose of storage, so that hysteresis effect due to change in temperature and possibility of leakage currents owing to moisture on the insulating materials between the terminals of the cell are avoided.
- Cells which are in general laboratory service should be frequently checked against each other. It is advisable to have three or more cells available in the laboratory, of which one or more are reserved solely for such checks and are not subject to the hazards of routine operations.

RESISTANCE STANDARDS AND CAPACITANCE STANDARDS

Resistance Standards

Resistance standards for d.c.. The resistor consists of a coil of platinum solver or maganin with non-inductively wound on metal former. Bifilar winding is used to obtain an almost non-inductive resistors.

Low Resistance Standards. Low resistance standards of value less than 1 Ω normally have to carry large currents. hence their heat-dissipating surface is large so that the temperature rise is kept low. These resistance standards are frequently used for potentiometer measurements where it is convenient to have a one-volt drop. All low resistance standards are of four terminal types.

Standard Resistances for A.C. ckts. In A.C. measurements it is absolute necessary that the standard resistors should be non reactive i.e. they should not have any inductance and capacitance.

Capacitance Standards

Primary Standards. Three types of construction have been used for primary standards:

(1) two concentric spheres

(2) two concentric cylinders

(3) two parallel plates with guard rings

Air capacitors, vacuum capacitor and gas filled capacitors come in this category.

Secondary Standards. Secondary standards and working standards of capacitance need not be provided with guard rings and their dimensions may not be accurately known as their value is fixed after comparison with primary standards. However, their capacitance should not change with time and in order to achieve this we should use material, which do not warp and so alter in dimensions.

Mica capacitor, solid dielectric capacitors, plastic film capacitors, ceramic capacitors etc come in this category.

PRIMARY STANDARDS OF TEMPERATURE & LUMINOUS INTENSITY

Temperature Standards

Thermodynamic temperature is one of the basic SI quantities whose unit is degree Kelvin. The thermo dynamic Kelvin scale is recognized as the basic scale to which all temperatures should be referred. The temperatures of the scale are designated as ^{0}K and denoted by the symbol T. The magnitude of degree Kelvin has been fixed by defining the thermodynamic temperature of the triple point of water at exactly 273.16^{0}K.

Since temperature measurements on the thermodynamic scale are inherently difficult, the Seventh General Conference of Weights and Measures adopted in 1927 , a practical scale which has been modified several times and is now called the “International Practical scale of Temperature”. The temperatures on this scale are designated as ^{0}C (degree Celsius) and denoted by the symbol t. The Celsius scale has two fundamental points the boiling point of water as 100^0 C and the triple point of water of 0.01^{0}C, both points are established at atmospheric pressure.

The primary “standard Thermometer” is the platinum resistance thermometer of special construction.

Luminous Intensity Standards

The primary standard of luminous intensity is a full radiator (black body or Plankian radiator at the temperature of solidification of platinum (2,042^{0}K approximately). The “candela” is then defined as one-sixtieth of luminous intensity per cm^2 of one full radiator.

Secondary standards of luminous intensity are special tungsten lamps calibrated against basic standards.

Units

UNIT

The result of a measurement of a physical quantity must be defined both in kind and magnitude. The standard measure of each kind of physical quantity is called a unit. Measurement implies comparison with a standard one.

ABSOLUTE UNIT

An absolute system of unit is defined as a system in which the various units are expressed in terms of small number of fundamental units. Absolute measurements do not compare the measured quantity with arbitrary units of the same type but made in terms of fundamental units.

FUNDAMENTAL UNIT

The fundamental units in mechanics are measures of length, mass and time. The sizes of fundamental units are arbitrary, and can be selected to fit a certain set of circumstances.

DERIVED UNIT

All other units, which can be expressed in terms of the fundamental units, are called derived units. Every unit originates from some physical law defining that unit. For example, the area (A) of a rectangle is proportional to its length(l) and breadth(b), or A = lb. If the meter has been chosen as the unit of length, then the area of a rectangle of 3m x 4 m is 12 m^2. Note that the numbers of measure are multiplied (3 x 4 = 12) as well as the units (m x m = m^2). The derived unit for area(A) is then the square meter(m^2).

SI UNITS

- **Length.** Metre (meter): m : Equal to 1650,763,73 wavelengths in vacuum.
- **Mass.** Kilogram: kg : Equal to mass of international prototype(platinum iridium cylinder) of mass.
- **Time.** Second: s : Duration of 9,192,631,770 periods of radians.
- **Electric current**. **Ampere (A)** . The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible cross-section and placed one meter apart in vacuum would produce between them a force equal to 2 x 10^{-7} Newtons per length.

- **Thermodynamic temperature. Kelvin (K)**. Fraction 1/273.16 of thermodynamic temperature of triple point of water.
- **Amount of substance. Mole (mol)**.Contains as many elementary entities as there are atoms in 0.012 kg of carbon 12.
- **Luminous intensity. Candela (cd).** Unit of luminous intensity, in a perpendicular direction, of a surface of 1/600,000 square meter of a black body at the temperature of freezing platinum under a pressure of 101.325 Newton per sq. meter.

IEEE STANDARDS

A slightly different type of standard is published and maintained by the Institute of Electrical and Electronics Engineers, IEEE, an engineering society headquartered in New York. These standards are not physical items that are available for comparison and checking of secondary standards but are standard procedures, nomenclature, definitions etc. These standards have been kept updated.

A large group of the IEEE standards is the standard test methods for testing, and evaluating various electronics systems and components. As an example, there is a standard method for testing and evaluating attenuators.

Another useful standard is the specifying of test equipment. The common laboratory oscilloscope becomes difficult to use when each manufacturer adopts a different arrangement of knobs and functions and, worst of all, different names for the same function. An IEEE standard addresses the laboratory oscilloscope and specifies the controls, functions etc, so that an oscilloscope operator does not have to reeducate himself for each oscilloscope he uses.

There are various standards concerning the safety of wiring for power plants, ships, industrials buildings etc.

Standard schematic and logic symbols are defined so that engineering drawings can be understood by all engineers.

Perhaps one of the most important standards is the IEEE 488 digital interface for programmable instrumentation for test and other equipment.

Errors

DIFFERENT TYPES OF ERRORS IN MEASUREMENT

- Gross errors
- Systematic errors
- Random errors

Gross errors

This class of errors mainly covers human mistakes in reading instruments and recording and calculating measurement results e.g. an experiment due to an oversight may read the temperature at 31.5^0C while the actual reading may be 21.5^0C.

Gross error may be of any amount and therefore their mathematical analysis is impossible.

Systematic errors

They can be classified into three categories :

Instrumental errors. These errors arise due to three main reasons.

- **Due to inherent shortcoming of the instrument**. These errors are inherent in instruments because of the mechanical structure. For example, if the spring of a permanent magnet instrument has become weak, the instrument will read high.
- **Misuse of instrument**. These errors are caused in measurements due to the fault of the operator than that of the instrument. For example , An experimenter may fail to adjust the zero of instrument.
- **Due to loading effects**. A well-calibrated voltmeter may give a misleading voltage when connected across a high resistance circuit. The same voltmeter when connected in a low resistance circuit may give a more dependable reading. Hence the voltmeter has a loading effect on the circuit.

Environmental errors. These are due to the measuring device including conditions surrounding the instrument. These may include temperature, pressure, humidity, dust etc.

Observational errors. There are many sources of observation errors. As an example the pointer of a voltmeter rests slightly above the surface of the scale. Thus an error on account of PARALLAX will be introduced unless the line of vision of the observer is exactly above the pointer.

Random Error

Experimental results show variation from one reading to another, even after all systematic errors have been accounted for. These errors are due to the multitude of small factors, which change or fluctuate from one measurement to another and are due surely to chance. The happening or disturbances about which we are unaware are called RANDOM and the errors caused by these happening is called RANDOM errors.

Example(AMIE W 93) :

The relative errors in measurement of power P, voltage V and current I are ± 0.5%, ± 1% , and ± 1% respectively. If power factor cosφ = P/VI (i) Calculate the relative error in power factor measurement (ii) also calculate the uncertainty in the power factor if the above error were specified as uncertainties.

Solution

$\text{Cos}\varphi = P/VI = P(VI)^{-1}$

Relative error in measurement of power factors

$= \pm [(\delta P/P) + (\delta V/V) + (\delta I/I)]$

$= \pm (0.5\% + 1\% + 1\%)$

$= \pm 2.5\%$

$\cos\varphi = P/VI$

partially differentiating w.r.t to P first

$\therefore \quad \partial(\cos\varphi)/\partial P = 1/VI$

partially differentiating w.r.t. to V

$\partial(\cos\varphi)/\partial V = -P/V^2I$

partially diff with respect to I

$\partial(\cos\varphi)/\partial I = -P/VI^2$

Resultant uncertainty

$w_x = \sqrt{} [(\partial\cos\varphi^2)/\partial P]w_p + [(\partial\cos\varphi)^2/\partial V]^2 w_v + [(\partial\cos\varphi)/\partial I)]^2 w_I$

w_p, w_v, w_I are uncertainties in power , voltage and current given respectively.

Putting values, we get

$w_x = \sqrt{} (1/VI)^2 w_v + (P/V^2I)^2 w_V + (P/VI)^2 w_I$

Put $\cos\varphi = P/VI$

$w_x = \sqrt{}(\cos\varphi)^2(w_p/P)^2 + \cos\varphi^2(w_V/V)^2 + (\cos\varphi)^2(w_I/I)^2$

% uncertainty in measurement of power factor is calculated by

$(w_x/\cos\varphi) \times 100$

$= (1/\cos\varphi)\sqrt{}(\cos\varphi)^2(w_p/P)^2 + \cos\varphi^2(w_V/V)^2 + (\cos\varphi)^2(w_I/I)^2 \times 100$

Putting various values

we get ± 1.5 % **Answer**

Example(AMIE W 94)

Three resistors have the following ratings :

$R_1 = 47\Omega \pm 4\%$, $R_2 = 65\Omega \pm 4\%$, $R_3 = 55\Omega \pm 4\%$

Determine the magnitude and limiting error in ohms and in percentage of the resistance of these resistances connected in series.

Solution

The value of the resistance is

$$R_1 = 47 \pm \frac{4}{100} \text{ x } 47 = 48.88\ \Omega$$

$$R_2 = 65 \pm \frac{4}{100} \text{ x } 65 = 6760\ \Omega$$

$$R_3 = 55 \pm \frac{4}{100} \text{ x } 55 = 5720\ \Omega$$

The limiting value of resultant resistance

$$R = (47 + 65 + 55) \pm (188 + 260 + 200) = 167 \pm 668$$

Magnitude of resistance obtained from series connection is 167Ω and error is ± 668 Ω.

$\therefore$ Percentage limiting error of series combination of resistances = $\pm \frac{668}{167} \text{ x}100 = \pm 4\ \%$.

Example(AMIE S96)

A voltmeter and an ammeter are to be used to determine the power dissipated in a resistor. Both the instruments are guaranteed to be accurate within ±1% at full-scale deflection. If the voltmeter reads 80V on its 150 V range and the ammeter reads 70mA on its 100mA range, determine the limiting error for the power calculation.

Solution

Power = VI

$\therefore$ The relative limiting error in the measurement of power is given by

$$\frac{\delta P}{P} = \pm\left\{\frac{\delta V}{V} + \frac{\delta I}{I}\right\}$$

$$\delta V = \frac{1}{100} \text{ x } 150 = 1.5$$

$$\delta I = \frac{1}{100} \text{ x } 100 = 1.0$$

$$P = 80 \text{ x } 70 \text{ x } 10^{-3} = 5600 \text{ x } 10^{-3} = 5.6 \text{ watts}$$

$$\therefore \quad \frac{\delta P}{5.6} = \pm\left\{\frac{1.5}{80} + \frac{1.0}{70}\right\} = \pm(0.01875 + 0.01423) = \pm\ 0.03298$$

$$\therefore \quad \delta P = \pm(0.03298) \text{ x } 5.6 = 0.1847$$

$\therefore$

Example (AMIE Winter2000)

The current passing through a resistor of 100 ± 0.2 Ω is 2.00 ± 0.01 A. Using the relationship $P = I^2R$, calculate the limiting error in the compound value of power dissipation.

Solution

Expressing the guaranteed limits of both current and resistance in percentages instead of units, we obtain

$$I = 2.00 \pm 0.01A = 2.00 \pm 0.5\%$$

$$R = 100 \pm 0.2\Omega = 100\ \Omega \pm 0.2\%$$

If the worst possible combination of errors is used, the limiting error in the power dissipation is ($P = I^2R$):

$$(2 \text{ x } 0.5\%) + 0.2\% = 1.2\%$$

Power dissipation should then be written as follows:

$$P = I^2R = (2.00)^2 \text{ x } 100 = 400 \text{ W} \pm 1.2\% = 400 \pm 4.8 \text{ W}$$

Example (AMIE Winter2001)

A voltmeter having a sensitivity of 1000 Ω/V, reads 100 V on its 150-V scale when connected across an unknown resistor in series with a millimeter.

When the millimeter reads 5 mA, calculate (a) apparent resistance of the unknown resistor (b) actual resistance of the unknown resistor (c) error due to the loading effect of the voltmeter.

Solution

(a) The total circuit resistance equals

$$R_T = \frac{V_T}{I_T} = \frac{100\,V}{5\,mA} = 20\,k\Omega$$

Neglecting the resistance of the millimeter, the value of the unknown resistor is R_x = 20 kΩ.

(b) The voltmeter resistance equals

$$R_V = 1000\frac{\Omega}{V}\text{x}150\,V = 150\,k\Omega$$

Since the voltmeter is in parallel with the unknown resistance, we can write

$$R_X = \frac{R_T R_V}{R_V - R_T} = \frac{20\text{x}150}{130} = 23.05k\Omega$$

(c) % error = $\frac{\text{actual} - \text{apparent}}{\text{actual}}\text{x}100\% = \frac{23.05 - 20}{23.05}\text{x}100\% = 13.23\%$

ASSIGNMENT

DIMENSIONS

Q.1. (AMIE W08, 12 marks): What is meant by the dimensions of a quantity? Derive the dimensions of potential difference in the electrostatic system in terms of mass, length and time. in the course of a calculation, an expression of the following form was arrived at:

$$I = E\left\{\frac{I}{Z_1} + \frac{j\omega M}{z_2}\left(\frac{1}{R} + \frac{C}{L}\right)\right\}$$

Show that there must have been an algebraic error and point out the term(s) which require correction.

Q.2. (AMIE W08, 8 marks): The electrical power in a circuit is proportional to the voltage and to the resistance of the circuit, each raised to some power. Determine these powers by the use of dimensions of quantities involved.

Answer: ML^2T^{-3}

Q.3. (AMIE S10, 8 marks): Derive the dimensions of (i) e.m.f. (ii) permeability (iii) resistivity (iv) electric flux density in L, M, T and I system of dimensions.

Answer: (i) $I^{-1}ML^2T^{-3}$ (ii) $I^{-2}MLT^{-2}$ (iii) $I^{-2}ML^3T^{-3}$ (iv) $I^{-1}MT^{-2}$

Q.4. (AMIE S12, 10 marks): Derive the dimensions of (i) permeability (ii) permittivity (iii) resistivity (iv) conductivity in L, M, T, I system of dimension.

Answer: permittivity = $I^2M^{-1}L^{-3}$; conductivity = 1/resistivity = $I^2M^{-1}L^{-3}T^3$

Q.5. (AMIE W12, 10 marks): Derive the dimensional equations for (i) current (ii) pole strength (iii) magnetic flux (iv) reluctance (iv) performance.

Q.6. (AMIE W13, 10 marks): Derive the dimensions of resistivity, magnetic flux, permittivity and electric field strength.

Q.7. (AMIE W13, 10 marks): Derive the dimensions of resistivity, magnetic flux, permittivity, and electric field strength.

Q.8. (AMIE W10, 10 marks): What do you mean by dimensions of quantity? The energy stored in a parallel plate capacitor per unit volume is given by $W = K\varepsilon^a v^b d^c$, where K = constant, ε = permittivity, v = voltage between plates, d = distance between plates. Find the values of a, b, c.

Answer: a = 1, b = 2, c = -2

Q.9. (AMIE W06, 10 marks): What are the advantages and disadvantages of Rationalized MKS system of units? What are the values of dielectric permittivity ε_0 and magnetic permittivity μ_0 of free space in Rationalized MKS system (RMKS). Verify the dimensional correctness of the equation. Power = voltage x current in RMKS system.

STANDARDS

Q.10. (AMIE W07, S13, 10 marks): What are different standards of measurements? What purpose do they serve?

Q.11. (AMIE W07, 9 marks): Discuss the following standards: (i) Voltage standards (ii) Resistance standards (iii) Capacitance standards

UNITS

Q.12. (AMIE W07, 5 marks): What are the advantages of SI units? State the SI units of the following quantities: MMF, Flux density, Frequency

Q.13. (AMIE S11, 10 marks): Define the terms "units", "absolute units", "fundamental units" and "derived units" with an example in each case.

ERRORS

Q.14. (AMIE S13, 10 marks): Explain about the errors in measurements.